Analysis of Crystal Oscillator

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Abstract

In this paper, analysis methods of a crystal oscillator are explained. The SPICE base method insures the $10^{-6} \sim 10^{-8}$ frequency resolution with short calculation time. The algebraic analysis methods presents the equations of negative resistance and equivalent capacitance, the frequency stability against power supply voltage, and the start-up times.

1 Introduction

A quartz crystal oscillator is a key device as a stable frequency source for communication equipments. Recently, there are urgent needs for improvement of its performance; low power operation, low power supply voltage operation, high frequency operation, phase noise reduction, miniaturization, etc. Precise and efficient analysis method of crystal oscillator is indispensable to meet these requirements. SPICE and its family simulators are widely used for crystal oscillator design[1]. Although SPICE is a powerful simulation tool, specific technique is effective for crystal oscillator simulation. We have been studied the effective and efficient analysis method and design technique of crystal oscillators. In this paper, we have outlined the analysis method using SPICE[2][3][4] and an algebraic approximate analysis method[5][6].

2 Analysis method using SPICE

Usually crystal oscillator is analyzed by transient analysis of SPICE. But its transient analysis requires large amount of calculation and obtained results are not sufficient in frequency resolution. Our approach is a kind of simplified harmonic balance technique.

2.1 Principle of analysis

Fig. 1 shows an equivalent circuit of a crystal oscillator. A crystal oscillator is composed of a crystal resonator and an active circuit. The oscillator circuit is usually designed to oscillate at the frequency where the crystal resonator is inductive. Therefore, the one-port impedance of the active circuit is expressed by the series circuit of a negative resistance R_i and an equivalent capacitance C_i. The negative resistance and the equivalent capacitance are the function of the frequency and the magnitude of the crystal current. On the other hand, the equivalent circuit of a crystal resonator is composed of the series resonance circuit of inductance L_1, capacitance C_1, resistance R_1, and the parallel capacitance C_0. At the steady state oscillation, the characteristics of a crystal is expressed by the series circuit of equivalent resistance R_e and inductive reactance X_L obtained by its equivalent circuit parameters. The steady state oscillation is attained when the negative resistance and equivalent capacitance of the active circuit balance the equivalent resistance and reactance of the resonator, respectively.

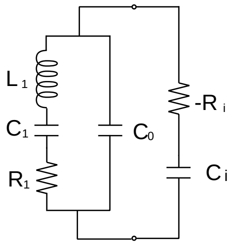

Figure 1: Equivalent circuit of crystal oscillator.

Usually, crystal current has only fundamental oscillation frequency, because the impedance of the crystal resonator becomes $\frac{1}{2\pi f C_0}$ at the harmonic oscillation frequency and is much bigger than the crystal equivalent resistance R_e. And the frequency change of the equivalent parameters of the active circuit is much smaller than that of the resonator. Therefore, when the current dependence of the equivalent parameters of the active circuit are calculated beforehand, the oscillation frequency and the crystal current can be calculated by these characteristics and the equivalent parameters of a crystal resonator.

2.2 Calculation method of negative resistance and equivalent capacitance

The negative resistance and the equivalent capacitance of the active circuit can be calculated by the following procedure; At first, the voltage response at the crystal terminals is calculated using the transient analysis, when the crystal resonator is replaced by the sinusoidal current source with magnitude I_x and frequency f in Fig. 1. Then, the magnitude V_x and phase ϕ of the fundamental component of the voltage response are calculated from the 1-period data of the steady state voltage response. The negative resistance and the equivalent capacitance are calculated using obtained V_x and phse ϕ and the crystal current I_x, by the following equations.

$$-R_i = \frac{V_x}{I_x}\cos\phi \qquad (1)$$

$$C_i = \frac{I_x}{2\pi f V_x \sin\phi} \qquad (2)$$

2.3 Calculation method of steady state characteristics

The crystal current and the oscillation frequency at the steady state can be calculated by following procedure; The crystal current dependence of the negative resistance and the equivalent capacitance are obtained at the power supply voltage V_{CC} and the crystal nominal frequency, as in Fig. 2. The steady state crystal current is obtained from the current value at the point A_1 where the magnitude of negative resistance agrees with the crystal equivalent resistance R_e. The steady state equivalent capacitance is obtained from the value of capacitance at the current level obtained above. The steady state oscillation frequency is calculated from the obtained equivalent capacitance and the crystal parameters. The frequency deviation caused by the power supply voltage variation can be calculated by similar calculation for the power supply voltage $V_{CC} + dV_{CC}$. By this method, the frequency deviation of 1ppm $\sim$ 0.01ppm can be estimated by calculation of the active impedance with the $10^{-3} \sim 10^{-4}$ resolution.

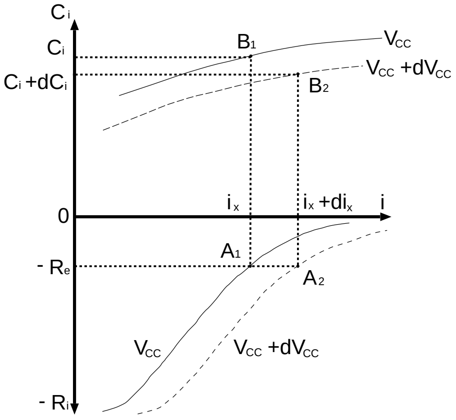

Figure 2: Crystal current dependence of negative resistance and equivalent capacitance.

2.3.1 Convergence

To obtain the negative resistance and the equivalent capacitance, the calculation of the transient analysis is necessary until the voltage response reaches the steady state. Judgment whether the response reaches the steady state is easily made by checking the variation of the period of voltage response cycle by cycle. Necessary cycles to reach the steady state is roughly proportional to the number of transistors used in the oscillator circuit. Although necessary cycles depend on the purpose of the calculation, 20 $\sim$ 50 cycles is sufficient for the colpitts crystal oscillator composed by single transistor.

2.4 Example

Fig. 3 shows the schematic diagram of the cascode crystal oscillator[7] widely used as a frequency source of mobile communication equipments. The negative resistance and the equivalent capacitance, and frequency stability against the power supply voltage are calculated and compared to the measurement.

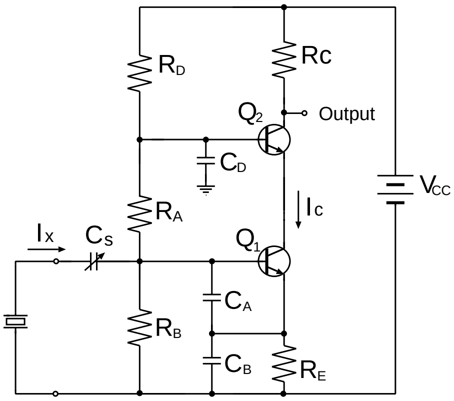

Figure 3: Cascode crystal oscillator

2.5 Negative resistance and equivalent capacitance

Fig. 4 shows the crystal current dependence of the negative resistance and the equivalent capacitance. The values at $I_x = 0$ were calculated by the small signal analysis (AC analysis). The calculated results agree with the measurement results within a several percent error. The margin of negative resistance for oscillation can be estimated from the difference between the values at the steady state current and null level.

2.5.1 Frequency stability

Fig. 5 shows the frequency deviation caused by the power supply voltage variation. Calculated results agree with the measurement within 10^{-7} order error in absolute values. The resolution of $10^{-6} \sim 10^{-8}$ can be obtained for the estimation of frequency change . Averaging is necessary for the estimation of frequency deviation in 10^{-8} resolution.

2.6 Automatic calculation and hybrid harmonic balance method

To obtain the crystal current dependence of the negative resistance and the equivalent capacitance, the transient analyses are made repeatedly by changing the driving current. This procedure can be programmed by C language or shell script. And the simulation is made automatically by changing the input file of SPICE.

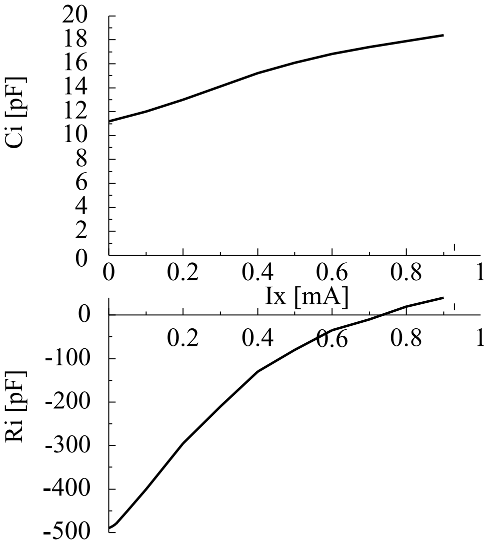

Figure 4: Crystal current dependence of negative resistance and equivalent capacitance

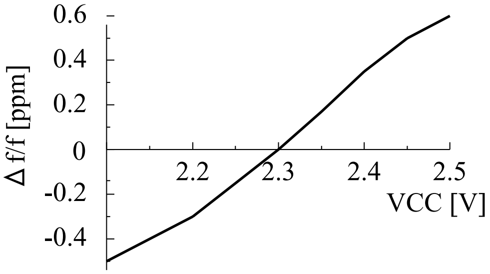

Figure 5: Frequency deviation caused by power supply voltage variation

In the case where the harmonic components of the crystal current are not negligible, as high frequency oscillator, the current source replaced crystal resonator must be parallel connection of the necessary harmonic component current sources. The voltage responses of the active circuit and the crystal resonator are calculated at each harmonic frequencies, and the magnitude and phase of each harmonic current sources are adjusted to obtain the agreement of the voltage responses. The obtained current values are the components of the steady state crystal current and the oscillation frequency can be calculated by the obtained equivalent parameters of the active circuit and the crystal parameters. This method is a modified harmonic balance method based on the time domain analysis and the frequency domain analysis[4].

Recently, sophisticated simulators base on the harmonic balance technique are available. ADS[8] and Micro wave office[9] are the examples of these simulators. These simulators also have the function of phse noise analysis.

3 Algebraic analysis method

Although simulator is a powerful tool for crystal oscillator design, it is difficult to grasp the relation between circuit performances and circuit parameters by simulation. We have derived the several approximate equations for the fundamental performances of a crystal oscillator.

3.1 Negative resistance and equivalent capacitance

The approximate expression of the negative resistance for the cascode oscillator shown in Fig. 3 is given by the following equations[6].

$$\begin{aligned} -R_i &= -\frac{g_m}{\omega^2 C'_A C_B} K\left(\frac{I_x}{\omega C'_A V_T}\right) + R_p \qquad (3) \\ R_p &= \frac{1}{R_{BE}(\omega C'_A)^2} + \frac{1}{R_E(\omega C_B)^2} + \frac{1}{R_{AB}(\omega C_t)^2} \qquad (4) \end{aligned}$$

where V_T is the thermal voltage, C'_A is the parallel capacitance of C_A and the large-signal base-emitter capacitance of transistor Q_1, C_t is parallel capacitance of C'_A and C_B, R_{AB} is the parallel resistance of R_A and R_B, R_{BE} is the large-signal base-emitter resistance.

g_m is the mutual conductance of transistor Q_1 at the large-signal collector DC current I_{C0} and is expressed by the following equations.

$$g_m = \frac{I_{CO}}{V_T} \qquad (5)$$

$$I_{C0} = \frac{\dfrac{R'_B V_A}{R_A + R_B} - V_{BE}}{\left(1 + \dfrac{1}{B_f}\right) R_E + \dfrac{R_A R'_B}{B_f (R_A + R'_B)}} \qquad (6)$$

$$\begin{aligned} V_{BE} &= \frac{1}{\dfrac{1}{V_T} + \dfrac{R_A + R'_B}{R'_B V_A}} \\ &\times \ln \frac{V_A}{I_S \left\{ \dfrac{(R_A + R'_B) R_E}{R'_B} \left(1 + \dfrac{1}{B_f}\right) + \dfrac{R_A}{B_f} \right\}} \qquad (7) \end{aligned}$$

where B_f, V_A, I_S are transistor model parameters, and R'_B is the parallel resistance of R_B and $B_f R_E$.
$K(v)$ is expressed as follows;

$$K(v) = \frac{2}{v} \frac{I_1(v)}{I_0(v)} \qquad (8)$$

where $I_0(v)$ and $I_1(v)$ are 0th and 1st order modified Bessel functions, respectively.

R_p is the equivalent loss component of the active circuit. The loaded Q of the oscillator is determined by the crystal equivalent resistance R_e and R_p.

The equivalent capacitance is given by the following equations.

$$C_i = C_{jBC} + \left(\frac{1}{C'_A} + \frac{\phi}{C_B}\right)^{-1} \qquad (9)$$

where,

$$\phi = 1 + \frac{g_m}{\omega C'_A} K\left(\frac{I_x}{\omega C'_A V_T}\right) \left(\frac{1}{\omega C'_A R_{BE}} + \frac{1}{\omega C_B R_E}\right) \qquad (10)$$

3.2 Frequency stability against power supply voltage variation

Frequency deviation caused by power supply voltage change is calculated by the deviation of equivalent capacitance of the active circuit using the following equation[5].

$$\frac{df/f}{dV_{CC}/V_{CC}} = \frac{C_i C_1}{2(C_i + C_0)^2} \frac{dC_i/C_i}{dV_{CC}/V_{CC}} \qquad (11)$$

The deviation of equivalent capacitance is calculated by the following equation.

$$
\begin{aligned}
\frac{dC_i/C_i}{dV_{CC}/V_{CC}} &= (\frac{1}{C_A}+\frac{1}{C_B})\frac{MJC}{VJC}\frac{CJC}{\left(1-\frac{V_{BC0}}{VJC}\right)^{1+MJC}}V_{BC0} \\
&+ \frac{1}{1+C_A/C_B}\frac{1}{C_A}\frac{MJE}{VJE}\frac{CJE}{\left(1-\frac{V_{BE}}{VJE}\right)^{1+MJE}} \\
&\times \frac{1}{1-\frac{V_{BE0}}{V_B}}\left\{V_T-\frac{1}{\frac{1}{2V_T k}-\frac{1}{V_B-B_{BE0}}}\right\}
\end{aligned}
\tag{12}
$$

where MJC,CJC,CJE,VJE are the transistor model parameters, V_{BC0}, V_{BE0}, V_B are the base-collector bias voltage, base-emitter bias voltage, base bias voltage, respectively.

k is the margin of the negative resistance: the ration of the negative resistances at the steady state oscillation and the small signal level.

Calculation of the frequency deviation are made and compared to the simulated values for the circuits with the parameters of Table 1. Tabel 2 shows the comparison of the frequency deviation between the algebraic approximate analysis and simulation. Sufficiently good agreement is obtained between the approximate analysis and simulation.

Table 1: Circuit parmeters

Parameter	No. 1	No. 2
$C_A = C_B$ $[pF]$	36.0	108.0
R_E $[k\Omega]$	18.0	6.0
$R_A = R_B$ $[k\Omega]$	128.0	42.7
L_1 $[mH]$	5.80	5.80
C_1 $[fF]$	11.9	11.9
R_1 $[\Omega]$	14.0	14.0
C_0 $[pF]$	2.84	2.84

3.3 Start-up Characteristics of Cascode Crystal Oscillator

On the design of cellular phones, the reduction of the start-up time of a crystal oscillator is required for reduction of the power consumption. We have derived the algebraic analysis method of start-up characteristics of the cascode crystal oscillator.

Table 2: Comparison of frequency stability estimated by approximate analysis and simulation

	frequency deviation $[ppm]$		Error
Circuit	Simulation	Analysis	[%]
No. 1	0.485	0.543	12
No. 2	0.063	0.070	11

The start-up time is defined by the time where the crystal current reaches the 90% of its steady state value. The approximate equation of start-up time is given by the following equation.

$$
\begin{aligned}
t_{90\%} &= \frac{R_D C_D(R_A+R'_B)}{R_A+R'_B+R_D}\left\{3-\ln\left(1-\frac{V_{BE}}{V_{Bmax}}\right)\right\} \\
&- \frac{2L_1}{R_{n0}-(R_p+R_1)} \\
&\times \left(\ln\frac{I_{x2}}{I_{x1}}+\ln\frac{R_1-R_i(I_{x1})}{R_1-R_i(I_{x2})}\right) \\
&+ \frac{2L_1}{R_p+R_1}\left\{\ln 10+\ln\left(1-2\frac{R_p+R_1}{R_{n0}}\right)\right\}
\end{aligned}
\tag{13}
$$

where V_{Bmax} is DC base voltage, R_{n0} is small signal negative resistance, I_{x1}, I_{x2} are the crystal boundary current used by approximate analysis.

The analysis of the start-up characteristics of the cascode oscillator is made and compared to the simulation. Tabel 3 shows the circuit parameters of the circuits. Fig. 6 shows the start-up characteristics. Tabe 4 shows the calculated results of start-up time. Sufficiently good agreement is obtained between the approximate analysis and simulation.

4 Summary

The analysis methods of a crystal oscillator are presented. SPICE base method gives high resolution with short simulation time. Approximate analysis methods can estimate the frequency stability and start-up time with sufficient accuracy. Presented methods are thought to be useful for crystal oscillator design.

Table 3: Circuit parmeters of cascode oscillator

Parameter	No. 1	No. 2
C_A $[pF]$	180	60
C_B $[pF]$	220	180
C_D $[nF]$	10	10
C_S $[pF]$	42.29	–
R_A $[k\Omega]$	68	20.0
R_B $[k\Omega]$	62	40.0
R_C $[k\Omega]$	1.8	0.5
R_D $[k\Omega]$	47	15.4
R_E $[k\Omega]$	5.1	1.9
Q_1, Q_2	2SC1359	2SC3585
L_1 $[mH]$	18.72	27.6
C_1 $[fF]$	13.54	5.607
R_1 $[\Omega]$	11.9	14.56
C_0 $[pF]$	2.75	1.52
VC_{CC} $[V]$	5.0	3.0
f $[MH_z]$	10	12.8

Table 4: Comparison of start-up time estimated by approximate analysis and simulation

	start-up time $[ms]$		Error
Circuit	simulation	analysis	[%]
No. 1	36.97	35.20	4.8
No. 2	11.14	11.42	2.5

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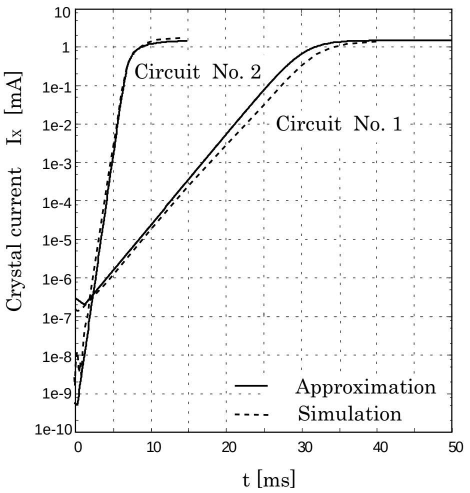

Figure 6: Start-up characteristics of crystal current calculated by approximate analysis and simulation.

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